Eur. Phys. J. C 39, 383–388 (2005)
Digital Object Identifier (DOI) 10.1140/epjc/s2004-02089-6

THE EUROPEAN
PHYSICAL JOURNAL C

Generation-changing interaction of sneutrinos in e^+e^- collisions

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Received: 6 October 2004 / Revised version: 14 November 2004 /
Published online: 14 January 2005 –

Abstract. The measurability of generation mixing is studied on pair production of sneutrinos in e^+e^- collisions and their subsequent decays into two different charged leptons e and μ with two lighter charginos. The analyses are made systematically in a general framework of the supersymmetric extension of the standard model. The production and decay process depends on the parameters of the chargino sector as well as those of the sneutrino sector. Although generation-changing interactions are severely constrained by radiative charged-lepton decays, sizable regions in the parameter space could still be explored at e^+e^- colliders in the near future.

PACS. 11.30.Hv, 12.15.Ff, 12.60.Jv, 14.80.Ly

1 Introduction

Mixing of quarks or leptons belonging to different generations is suggestive. It could give a clue to underlying theories for the generations. To this end, we should get phenomenological information about the mixing as much as possible. Until recently most experimental data for the mixing were on the quarks, though data on the leptons are accumulating from the neutrino oscillations [1]. As a result, for instance, an examination of grand unified theories can now be made through comparison of the mixing for quarks and leptons [2].

The supersymmetric standard model (SSM) is considered a plausible candidate for physics beyond the standard model (SM). This model includes superpartners of quarks and leptons, which could also be mixed among different generations. If the SSM is indeed the extension of the SM, information about generation mixing of these supersymmetric particles will also help us to gain deep insight into the generations. In future experiments, therefore, the study of generation mixing in the SSM may well be an important subject. It has been shown that a pair of different charged leptons, especially τ and μ or e, could be produced at detectable rates in e^+e^- collisions, in a certain scenario for the slepton generation mixing [3] or with the general mixing structure [4].

In this article we study the generation-changing interaction of charged leptons, sneutrinos and charginos without assuming a specific scenario for generation mixing. This interaction can create two different charged leptons with two charginos in e^+e^- annihilation through the sneutrinos on mass-shell, which may be measured in near future experiments. We obtain the cross section of this process within the general framework of the SSM. The same interaction, on the other hand, induces radiative charged-lepton decays, which are constrained by non-observation in experiments to date [5]. Under these constraints, concentrating on the production of e and μ with two lighter charginos, systematic analyses are performed to discuss the possibility of measuring the interaction at e^+e^- colliders. Particular attention is paid to the dependences on various SSM parameters. It is shown that there are sizable regions of the parameter space which could be examined in the future experiments.

In e^+e^- annihilation a pair of sneutrinos $\tilde{\nu}_a$ are created if kinematically allowed: $e^+e^- \to \tilde{\nu}_a\tilde{\nu}_b^*$, where the indices a and b represent the generations. Assuming that the sneutrinos are heavier than some of the charginos ω_i, the sneutrino can decay into a charged lepton and a chargino $\tilde{\nu}_a^{(*)} \to l_\alpha^{-(+)}\omega_i^{+(-)}$, with α representing the generation. The generation mixing could be measured by tagging the charged leptons. However, it is suggested by both theoretical considerations and experimental constraints derived from radiative charged-lepton decays that some sneutrino masses are highly degenerate. Then, the generation mixing has to be analyzed without specifying the sneutrino generations [6]. Our analyses are made generically under the assumption that the sneutrinos of three generations are produced at the same collision energy and all the sneutrinos can yield a charged lepton of any generation.

This paper is organized as follows. In Sect. 2, the interactions of sneutrinos relevant to their pair production and dominant two-body decays are summarized. In Sect. 3 the cross section is given for the production of two charged leptons and two charginos mediated by real sneutrinos in e^+e^- annihilation. In Sect. 4, numerical analyses for the production of e and μ with two lighter charginos are performed, discussing the measurability of generation mixing. A summary is given in Sect. 5.

2 Interactions

The mass eigenstates of the leptons or the sneutrinos are generally not the same as their interaction eigenstates. Assuming that no superfield for a right-handed neutrino exists at the electroweak energy scale, the sneutrinos have a 3×3 hermitian mass-squared matrix $\tilde{M}_\nu^2$. The generation mixing of the sneutrinos is traced back to soft supersymmetry-breaking terms. The mass matrix M_l for the charged leptons is proportional to the coefficient matrix of the Higgs couplings. For the left-handed neutrinos, we assume a Majorana mass matrix M_ν. These mass matrices for the leptons are 3×3 and non-diagonal. The mass eigenstates are obtained by diagonalizing these matrices as follows:

$$\tilde{U}_\nu^\dagger \tilde{M}_\nu^2 \tilde{U}_\nu = \mathrm{diag}(M_{\tilde{\nu}_1}^2, M_{\tilde{\nu}_2}^2, M_{\tilde{\nu}_3}^2), \tag{1}$$

$$U_{l\mathrm{R}}^\dagger M_l U_{l\mathrm{L}} = \mathrm{diag}(m_{l_1}, m_{l_2}, m_{l_3}), \tag{2}$$

$$U_\nu^\mathrm{T} M_\nu U_\nu = \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}), \tag{3}$$

where $\tilde{U}_\nu$, $U_{l\mathrm{R}}$, $U_{l\mathrm{L}}$, and U_ν are unitary matrices. The masses of the charged leptons e, μ, and τ are denoted by m_{l_1}, m_{l_2}, and m_{l_3}, respectively. For the parameters which describe the sneutrino mass-squared matrix, we take the mass eigenvalues $M_{\tilde{\nu}_a}$ $(a = 1, 2, 3)$ and the unitary matrix $\tilde{U}_\nu$.

The charginos ω_i $(i = 1, 2)$ and the neutralinos χ_n $(n = 1$–$4)$ are the mass eigenstates for the $SU(2)\times U(1)$ gauginos and higgsinos. The mass matrices M^- and M^0 for the charginos and the neutralinos, respectively, are given by

$$M^- = \begin{pmatrix} \tilde{m}_2 & -\frac{1}{\sqrt{2}}gv_1 \\ -\frac{1}{\sqrt{2}}gv_2 & m_H \end{pmatrix}, \tag{4}$$

$$M^0 = \begin{pmatrix} \tilde{m}_1 & 0 & \frac{1}{2}g'v_1 & -\frac{1}{2}g'v_2 \\ 0 & \tilde{m}_2 & -\frac{1}{2}gv_1 & \frac{1}{2}gv_2 \\ \frac{1}{2}g'v_1 & -\frac{1}{2}gv_1 & 0 & -m_H \\ -\frac{1}{2}g'v_2 & \frac{1}{2}gv_2 & -m_H & 0 \end{pmatrix}. \tag{5}$$

Here, $\tilde{m}_1$ and $\tilde{m}_2$ stand for the gaugino mass parameters for respectively $U(1)$ and $SU(2)$, for which we assume the relation $\tilde{m}_1 = (5/3)\tan^2\theta_\mathrm{W}\tilde{m}_2$ suggested by the $SU(5)$ grand unified theory. The higgsino mass parameter is denoted by m_H. The vacuum expectation values of the Higgs bosons with hypercharges $-1/2$ and $1/2$ are expressed by v_1 and v_2, respectively, the ratio v_2/v_1 being denoted by $\tan\beta$. These mass matrices are diagonalized to give mass eigenstates as

$$C_\mathrm{R}^\dagger M^- C_\mathrm{L} = \mathrm{diag}(m_{\omega_1}, m_{\omega_2}), \tag{6}$$
$$m_{\omega_1} < m_{\omega_2},$$

$$N^\mathrm{T} M^0 N = \mathrm{diag}(m_{\chi_1}, m_{\chi_2}, m_{\chi_3}, m_{\chi_4}), \tag{7}$$
$$m_{\chi_1} < m_{\chi_2} < m_{\chi_3} < m_{\chi_4},$$

where C_R, C_L, and N are unitary matrices.

We express the interaction Lagrangians for the sneutrinos $\tilde{\nu}_a$ in terms of particle mass eigenstates. The Lagrangian for charged leptons, sneutrinos, and charginos is given by

$$\mathcal{L} = \mathrm{i}\frac{g}{\sqrt{2}}(V_C)_{a\alpha}\tilde{\nu}_a^\dagger\overline{\omega_i} \times\left[\sqrt{2}C_{\mathrm{R}1i}^*\left(\frac{1-\gamma_5}{2}\right) + C_{\mathrm{L}2i}^*\frac{m_{l_\alpha}}{\cos\beta M_W}\left(\frac{1+\gamma_5}{2}\right)\right] l_\alpha + \mathrm{H.c.}, \tag{8}$$

with $V_C = \tilde{U}_\nu^\dagger U_{l\mathrm{L}}$. The generation mixing is described by V_C, which is a 3×3 unitary matrix. For the independent physical parameters of V_C three mixing angles and one complex phase can be taken, the other complex phases being left out by redefinition of the particle fields. We adopt the parametrization in the standard form for the Cabibbo–Kobayashi–Maskawa matrix:

$$V_C = \tag{9}$$
$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\mathrm{e}^{-\mathrm{i}\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta} & c_{23}c_{13} \end{pmatrix},$$

with $c_{ab} = \cos\theta_{ab}$ and $s_{ab} = \sin\theta_{ab}$. Without loss of generality, the angles θ_{12}, θ_{23}, and θ_{13} can be put in the first quadrant. The Lagrangian for neutrinos, sneutrinos, and neutralinos is given by

$$\mathcal{L} = \mathrm{i}\frac{g}{\sqrt{2}}(V_N)_{a\alpha}(-\tan\theta_\mathrm{W}N_{1n} + N_{2n})\tilde{\nu}_a^\dagger\overline{\chi_n}\left(\frac{1-\gamma_5}{2}\right)\nu_\alpha + \mathrm{H.c}, \tag{10}$$

with $V_N = \tilde{U}_\nu^\dagger U_\nu$. The 3×3 unitary matrix V_N describes the generation mixing. The number of its physical parameters is six, though a definite parametrization is not necessary for our analyses. Finally the coupling with the Z-boson is expressed by the Lagrangian

$$\mathcal{L} = -\mathrm{i}\frac{\sqrt{g^2+g'^2}}{2}\left(\tilde{\nu}_a^*\partial^\mu\tilde{\nu}_a - \partial^\mu\tilde{\nu}_a^*\tilde{\nu}_a\right)Z_\mu, \tag{11}$$

which does not cause generation-changing interaction.

3 Cross section

A pair of sneutrinos are created in e^+e^- annihilation by the Z-boson and chargino exchange diagrams. We assume that the masses of the sneutrinos are almost degenerate among three generations. The electron and positron couple to all the sneutrinos through charginos as shown in (8). Therefore, a pair of sneutrinos in any combination of generations can be produced at a collision energy above the threshold. The sneutrino dominantly decays into a charged lepton and a chargino or into a neutrino and a neutralino. These leptons can also belong to any generation, owing to the interactions in (8) and (10).

We give the cross section for $e^+e^- \to l_\alpha^- l_\beta^+ \omega_i^+ \omega_j^-$ mediated by the sneutrinos on mass-shell. In calculating this cross section the products of the sneutrino propagators are involved. We make an approximation:

$$\frac{1}{(q^2 - M_{\tilde{\nu}_a}^2 + \mathrm{i}M_{\tilde{\nu}_a}\Gamma_{\tilde{\nu}_a})(q^2 - M_{\tilde{\nu}_b}^2 - \mathrm{i}M_{\tilde{\nu}_b}\Gamma_{\tilde{\nu}_b})}$$

$$= \frac{\pi\left\{\delta(q^2-M_{\tilde\nu_a}^2)+\delta(q^2-M_{\tilde\nu_b}^2)\right\}}{2(1+\mathrm{i}x_{ab})(\overline{M_{\tilde\nu}\Gamma_{\tilde\nu}})_{ab}}, \tag{12}$$

$$(\overline{M_{\tilde\nu}\Gamma_{\tilde\nu}})_{ab} = \frac{M_{\tilde\nu_a}\Gamma_{\tilde\nu_a}+M_{\tilde\nu_b}\Gamma_{\tilde\nu_b}}{2},$$

$$x_{ab} = \frac{M_{\tilde\nu_a}^2-M_{\tilde\nu_b}^2}{M_{\tilde\nu_a}\Gamma_{\tilde\nu_a}+M_{\tilde\nu_b}\Gamma_{\tilde\nu_b}},$$

where $\Gamma_{\tilde\nu_a}$ denotes the total decay width of the sneutrino $\tilde\nu_a$. The relations $M_{\tilde\nu_a} \gg \Gamma_{\tilde\nu_a}$, $M_{\tilde\nu_b} \gg \Gamma_{\tilde\nu_b}$, and $|M_{\tilde\nu_a}^2 - M_{\tilde\nu_b}^2| \ll M_{\tilde\nu_a}^2 \approx M_{\tilde\nu_b}^2$ have been assumed. The cross section is written as

$$\begin{aligned}
&\sigma(e^+e^- \to l_\alpha^- l_\beta^+ \omega_i^+ \omega_j^-) \\
&= \sum_{a,a',b,b'} \frac{(V_C^\dagger)_{\alpha a}(V_C)_{a'\alpha}(V_C)_{b\beta}(V_C^\dagger)_{\beta b'}}{4(1+\mathrm{i}x_{aa'})(1+\mathrm{i}x_{bb'})(\overline{M_{\tilde\nu}\Gamma_{\tilde\nu}})_{aa'}(\overline{M_{\tilde\nu}\Gamma_{\tilde\nu}})_{bb'}} \\
&\quad\times\Big\{\tilde\sigma_{aa'bb'}(M_{\tilde\nu_a},M_{\tilde\nu_b})M_{\tilde\nu_a}M_{\tilde\nu_b}\tilde\Gamma(\tilde\nu_a\to l_\alpha\omega_i) \\
&\qquad\times\tilde\Gamma(\tilde\nu_b\to l_\beta\omega_j) \\
&\quad+\tilde\sigma_{aa'bb'}(M_{\tilde\nu_a},M_{\tilde\nu_{b'}})M_{\tilde\nu_a}M_{\tilde\nu_{b'}}\tilde\Gamma(\tilde\nu_a\to l_\alpha\omega_i) \\
&\qquad\times\tilde\Gamma(\tilde\nu_{b'}\to l_\beta\omega_j) \\
&\quad+\tilde\sigma_{aa'bb'}(M_{\tilde\nu_{a'}},M_{\tilde\nu_b})M_{\tilde\nu_{a'}}M_{\tilde\nu_b}\tilde\Gamma(\tilde\nu_{a'}\to l_\alpha\omega_i) \\
&\qquad\times\tilde\Gamma(\tilde\nu_b\to l_\beta\omega_j) \\
&\quad+\tilde\sigma_{aa'bb'}(M_{\tilde\nu_{a'}},M_{\tilde\nu_{b'}})M_{\tilde\nu_{a'}}M_{\tilde\nu_{b'}}\tilde\Gamma(\tilde\nu_{a'}\to l_\alpha\omega_i) \\
&\qquad\times\tilde\Gamma(\tilde\nu_{b'}\to l_\beta\omega_j)\Big\},
\end{aligned} \tag{13}$$

where the summation for each index is done over three generations. Expressing the total energy at the center-of-mass frame by $\sqrt{s}$, the factor $\tilde\sigma_{aa'bb'}(M_{\tilde\nu_a},M_{\tilde\nu_b})$ is defined by

$$\begin{aligned}
&\tilde\sigma_{aa'bb'}(M_{\tilde\nu_a},M_{\tilde\nu_b}) = \frac{g^4}{64\pi s^2}\int_{t_-}^{t_+}\mathrm{d}t\left(tu-M_{\tilde\nu_a}^2M_{\tilde\nu_b}^2\right) \\
&\Bigg[\delta_{ab}\delta_{a'b'}\left\{\left(\frac{-1+2\sin^2\theta_\mathrm{W}}{2\cos^2\theta_\mathrm{W}}\right)^2+\tan^4\theta_\mathrm{W}\right\} \\
&\quad\times\frac{1}{(s-M_Z^2)^2} \\
&\quad+(V_C)_{a1}(V_C^\dagger)_{1b}(V_C^\dagger)_{1a'}(V_C)_{b'1} \\
&\quad\times\sum_{k,l}\frac{|C_{\mathrm{R}1k}|^2|C_{\mathrm{R}1l}|^2}{(t-m_{\omega_k}^2)(t-m_{\omega_l}^2)} \\
&\quad+\left\{\delta_{ab}(V_C^\dagger)_{1a'}(V_C)_{b'1}+(V_C)_{a1}(V_C^\dagger)_{1b}\delta_{a'b'}\right\} \\
&\quad\times\frac{-1+2\sin^2\theta_\mathrm{W}}{2\cos^2\theta_\mathrm{W}}\sum_k\frac{|C_{\mathrm{R}1k}|^2}{(t-m_{\omega_k}^2)(s-M_Z^2)}\Bigg],
\end{aligned} \tag{14}$$

$$u = -s-t+M_{\tilde\nu_a}^2+M_{\tilde\nu_b}^2,$$

$$t_\pm = \frac{1}{2}\Big\{M_{\tilde\nu_a}^2+M_{\tilde\nu_b}^2-s \pm\sqrt{s^2-2(M_{\tilde\nu_a}^2+M_{\tilde\nu_b}^2)s+(M_{\tilde\nu_a}^2-M_{\tilde\nu_b}^2)^2}\Big\},$$

which arises from the sneutrino pair production. The sneutrino decay yields the factor $\tilde\Gamma(\tilde\nu_a\to l_\alpha\omega_i)$, which is defined by

$$\begin{aligned}
&\tilde\Gamma(\tilde\nu_a\to l_\alpha\omega_i) = \frac{g^2}{16\pi} \\
&\times M_{\tilde\nu_a}\sqrt{\left\{1-\frac{(m_{l_\alpha}+m_{\omega_i})^2}{M_{\tilde\nu_a}^2}\right\}\left\{1-\frac{(m_{l_\alpha}-m_{\omega_i})^2}{M_{\tilde\nu_a}^2}\right\}} \\
&\times\Bigg\{\left(|C_{\mathrm{R}1i}|^2+|C_{\mathrm{L}2i}|^2\frac{m_{l_\alpha}^2}{2\cos^2\beta M_W^2}\right)\left(1-\frac{m_{l_\alpha}^2+m_{\omega_i}^2}{M_{\tilde\nu_a}^2}\right) \\
&-\mathrm{Re}\,[C_{\mathrm{R}1i}C_{\mathrm{L}2i}^*]\frac{2\sqrt{2}m_{l_\alpha}^2m_{\omega_i}}{\cos\beta M_W M_{\tilde\nu_a}^2}\Bigg\}.
\end{aligned} \tag{15}$$

The total width of the sneutrino is approximately determined by the two-body decays,

$$\Gamma_{\tilde\nu_a} = \sum_{\alpha,i}\Gamma(\tilde\nu_a\to l_\alpha^-\omega_i^+)+\sum_{\alpha,n}\Gamma(\tilde\nu_a\to\nu_\alpha\chi_n), \tag{16}$$

$$\Gamma(\tilde\nu_a\to l_\alpha^-\omega_i^+) = |(V_C)_{a\alpha}|^2\tilde\Gamma(\tilde\nu_a\to l_\alpha\omega_i),$$

$$\begin{aligned}
&\Gamma(\tilde\nu_a\to\nu_\alpha\chi_n) = |(V_N)_{a\alpha}|^2\frac{g^2}{32\pi}M_{\tilde\nu_a} \\
&\qquad\times|-\tan\theta_\mathrm{W}N_{1n}+N_{2n}|^2\left(1-\frac{m_{\chi_n}^2}{M_{\tilde\nu_a}^2}\right)^2,
\end{aligned}$$

where the masses of the neutrinos have been neglected. Since the equality $\sum_\alpha|(V_N)_{a\alpha}|^2 = 1$ holds, the mixing matrix V_N need not to be specified for obtaining $\Gamma_{\tilde\nu_a}$.

The radiative charged-lepton decays $\mu\to e\gamma$, $\tau\to e\gamma$, and $\tau\to\mu\gamma$ are induced at one-loop level by the exchange of charginos and sneutrinos. For the calculation of these decay widths, we refer to the appendix of [7]. Applying the given formulae to the interaction in (8), the decay widths are obtained straightforwardly. The radiative decays are also generated by one-loop diagrams with charged sleptons and neutralinos. However, these contributions are generally smaller than those of the chargino–sneutrino diagrams, so that we neglect the former ones.

4 Numerical analyses

Specializing in the production and decay process $e^+e^-\to\sum_{a,b}\tilde\nu_a\tilde\nu_b^*\to e^-\mu^+\omega_1^+\omega_1^-$, we numerically discuss its cross section. The intermediate sneutrinos are on mass-shell, belonging to any generations. This process shows distinctive final states. The primary leptons e and μ are produced by two-body decays. Their energies are large, unless the mass of the lighter chargino is close to the sneutrino masses. In addition, they have approximately the same value and are monochromatic in each rest frame of the decaying sneutrino. The chargino yields, by three-body decays, two

Table 1. The parameter values for the charginos and neutralinos assumed in Figs. 1–5. The unit of mass is GeV

	(a)	(b)	(c)	(d)
$\tan\beta$	5	5	5	10
$\tilde{m}_2$	200	150	250	200
m_H	200	250	150	200
m_{ω_1}	138	121	121	144
m_{ω_2}	272	288	288	269
m_{χ_1}	83	65	88	86
m_{χ_2}	146	124	149	149
m_{χ_3}	207	257	157	209
m_{χ_4}	273	289	289	269

quarks or two leptons with a neutralino. Therefore, the final states involve, in each hemisphere, one charged lepton with a large energy of flat distribution and two jets or one charged lepton, together with large missing energy-momentum. The detection of the process will not be difficult. The collider energy is assumed to be $\sqrt{s} = 500\,\text{GeV}$ throughout the analyses.

The cross section under discussion depends on the mass differences of the sneutrinos, as the radiative charged-lepton decays do so. In Figs. 1, 2, and 3 we show, in the $M_{\tilde{\nu}_2}$–$M_{\tilde{\nu}_3}$ plane, the cross section for the regions allowed by the constraints from the radiative charged-lepton decays. The mass of $\tilde{\nu}_1$ is fixed at $M_{\tilde{\nu}_1} = 200\,\text{GeV}$. The mixing angles and complex phase of V_C are put at $\theta_{12} = \theta_{23} = \theta_{13} = 0.02$ and $\delta = \pi/4$. The parameter values for the charginos and neutralinos are listed in Table 1, the sets (a), (b), and (c) corresponding to the Figs. 1, 2, and 3, respectively. The resultant masses of the charginos and neutralinos are also given in the table. The cross section has a value $\sigma < 0.05\,\text{fb}$ in the light shaded regions, and in the dark shaded regions $0.05\,\text{fb} < \sigma < 0.07\,\text{fb}$, $0.05\,\text{fb} < \sigma < 0.1\,\text{fb}$, and $0.05\,\text{fb} < \sigma < 0.06\,\text{fb}$ for Figs. 1, 2, and 3, respectively. Unshaded regions are excluded by the radiative charged-lepton decays.

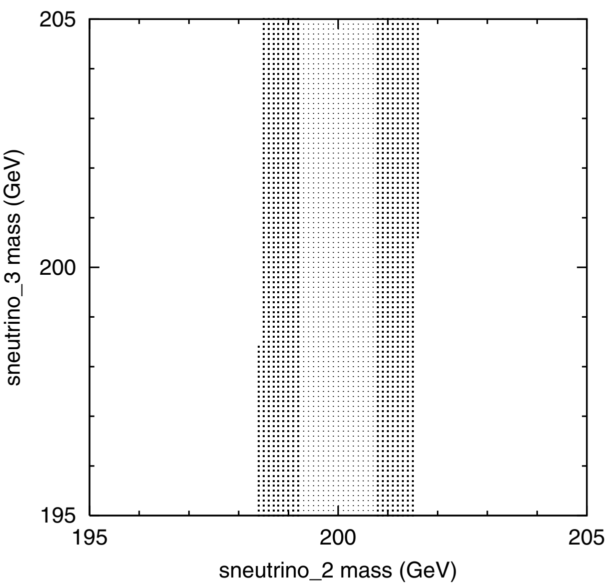

Fig. 1. The cross section for the parameter set (a) listed in Table 1: $\sigma < 0.05\,\text{fb}$ in the light shaded region and $0.05\,\text{fb} < \sigma < 0.07\,\text{fb}$ in the dark shaded region. $M_{\tilde{\nu}_1} = 200\,\text{GeV}$, $\theta_{12} = \theta_{23} = \theta_{13} = 0.02$, and $\delta = \pi/4$

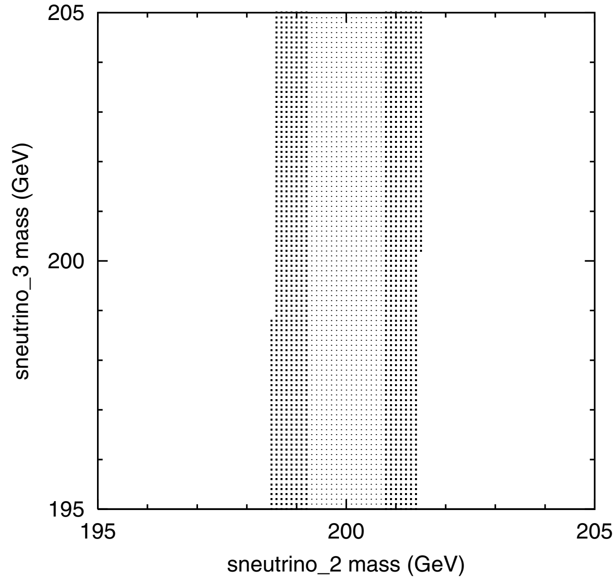

Fig. 2. The cross section for the parameter set (b) listed in Table 1: $\sigma < 0.05\,\text{fb}$ in the light shaded region and $0.05\,\text{fb} < \sigma < 0.1\,\text{fb}$ in the dark shaded region. $M_{\tilde{\nu}_1} = 200\,\text{GeV}$, $\theta_{12} = \theta_{23} = \theta_{13} = 0.02$, and $\delta = \pi/4$

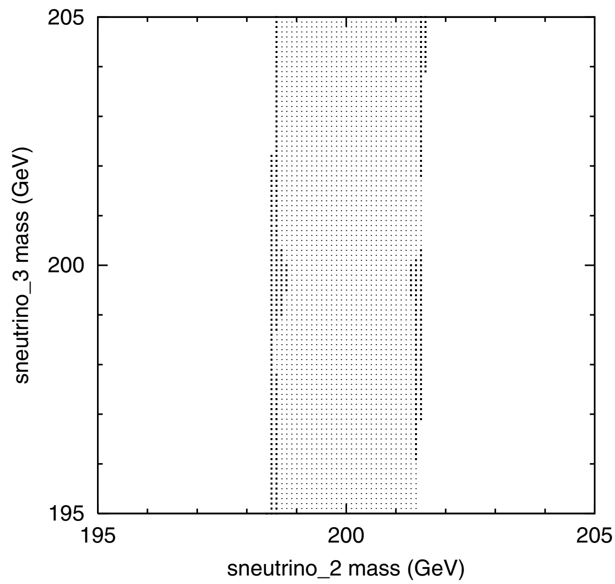

Fig. 3. The cross section for the parameter set (c) listed in Table 1: $\sigma < 0.05\,\text{fb}$ in the light shaded region and $0.05\,\text{fb} < \sigma < 0.06\,\text{fb}$ in the dark shaded region. $M_{\tilde{\nu}_1} = 200\,\text{GeV}$, $\theta_{12} = \theta_{23} = \theta_{13} = 0.02$, and $\delta = \pi/4$

As the mass difference between $\tilde{\nu}_1$ and $\tilde{\nu}_2$ becomes large, the cross section increases. The decay width of $\mu \to e\gamma$ also increases and becomes too large in the outside of the shaded region. The mass of $\tilde{\nu}_3$ does not sensitively affect the cross section nor the decay width. The constraints from the decays $\tau \to e\gamma$ and $\tau \to \mu\gamma$ are not very stringent, so that

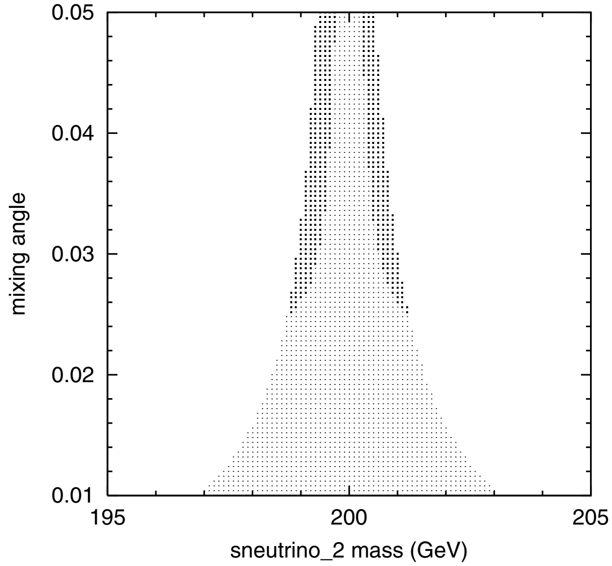

Fig. 4. The cross section for the parameter set (a) listed in Table 1: $\sigma < 0.1\,\mathrm{fb}$ in the light shaded region and $0.1\,\mathrm{fb} < \sigma < 0.3\,\mathrm{fb}$ in the dark shaded region. $M_{\tilde{\nu}_1} = 200\,\mathrm{GeV}$, $M_{\tilde{\nu}_3} = 198\,\mathrm{GeV}$, and $\delta = \pi/4$

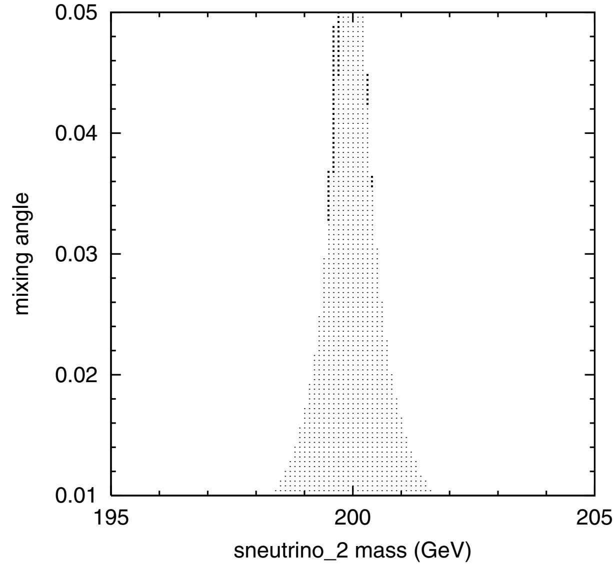

Fig. 5. The cross section for the parameter set (d) listed in Table 1: $\sigma < 0.1\,\mathrm{fb}$ in the light shaded region and $0.1\,\mathrm{fb} < \sigma < 0.2\,\mathrm{fb}$ in the dark shaded region. $M_{\tilde{\nu}_1} = 200\,\mathrm{GeV}$, $M_{\tilde{\nu}_3} = 198\,\mathrm{GeV}$, and $\delta = \pi/4$

large mass differences between $\tilde{\nu}_1$ and $\tilde{\nu}_3$ and between $\tilde{\nu}_2$ and $\tilde{\nu}_3$ are allowed. Comparing three parameter choices for $\tilde{m}_2$ and m_H of the sets (a), (b), and (c), we can see that the mass ranges of $\tilde{\nu}_2$ allowed by the radiative charged-lepton decays are not much different from each other. However, the cross section varies manifestly with these parameters, depending on the relative magnitudes of $\tilde{m}_2$ and m_H. For a smaller magnitude of $\tilde{m}_2/m_H$, the $SU(2)$-gaugino component of the lighter chargino is larger. Then, the cross section of $e^+e^- \to \tilde{\nu}_a\tilde{\nu}_b^*$ and thus that of $e^+e^- \to e^-\mu^+\omega_1^+\omega_1^-$ increase.

The dependences of the cross section on the mixing angles of V_C and $\tan\beta$ are shown in Figs. 4 and 5. Assuming the equality $\theta_{12} = \theta_{23} = \theta_{13}$ $(\equiv \theta)$ with $\delta = \pi/4$ for simplicity, the cross section is shown in the $M_{\tilde{\nu}_2}$–θ plane for the regions consistent with the radiative charged-lepton decays. The masses of $\tilde{\nu}_1$ and $\tilde{\nu}_3$ are fixed at $M_{\tilde{\nu}_1} = 200\,\mathrm{GeV}$ and $M_{\tilde{\nu}_3} = 198\,\mathrm{GeV}$. The parameter values for the charginos and neutralinos are given by the sets (a) and (d) in Table 1 for Figs. 4 and 5, respectively. The cross section has a value $\sigma < 0.1\,\mathrm{fb}$ in the light shaded regions, and in the dark shaded regions $0.1\,\mathrm{fb} < \sigma < 0.3\,\mathrm{fb}$ and $0.1\,\mathrm{fb} < \sigma < 0.2\,\mathrm{fb}$ for Figs. 4 and 5, respectively. As the mixing angle increases, the allowed range for the mass difference between $\tilde{\nu}_1$ and $\tilde{\nu}_2$ becomes narrow, while the cross section becomes large. These angle dependences are primarily determined by θ_{12}. Different values for θ_{23} and θ_{13} do not alter much the cross section and the allowed mass difference, as long as these mixing angles are small. For a larger value of $\tan\beta$, the allowed region becomes small, though the cross section does not vary much with it.

Under the constraints from the presently available experiments, the cross section of $e^+e^- \to e^-\mu^+\omega_1^+\omega_1^-$ is larger than $0.05\,\mathrm{fb}$ in sizable regions of the parameter space. It should be noted that there also exists a charge conjugate process which has the same cross section. For the integrated luminosity $100\,\mathrm{fb}^{-1}$, more than ten events are expected there. Although possible backgrounds have to be taken into account for estimating realistically available events, a number of the order of ten would not be insufficient for detection in near future experiments.

We now comment on possible background processes. The two leptons e and μ with missing energy-momentum can be produced by the $\tau^+\tau^-$ pair production and their subsequent leptonic decays, which is due to the generation-conserving interaction of the SM. This pair production occurs as e.g. $e^+e^- \to ZW^+W^-$, $Z \to \tau^+\tau^-$ in the SM and $e^+e^- \to \tilde{\nu}_3\tilde{\nu}_3^* \to \tau^-\tau^+\omega_1^+\omega_1^-$ in the SSM. The final states of these processes are similar to those of the discussed generation-changing process. However, there are wide differences in magnitude and distribution of the produced charged-lepton energy between the decays of τ and $\tilde{\nu}_a$. Appropriate energy cuts will be useful to reduce the backgrounds. Next, suppose that the masses of the charged sleptons are not much different from the sneutrino masses and, in addition, larger than the mass of the second lightest neutralino. Then, a pair of charged sleptons are produced and can decay into two charged leptons with two second lightest neutralinos, $e^+e^- \to \tilde{l}_a\tilde{l}_b^* \to l_\alpha^- l_\beta^+ \chi_2\chi_2$. Owing to possible generation-changing interaction of charged leptons, charged sleptons, and neutralinos, these two leptons could belong to different generations, such as e and μ. If the second lightest neutralino decays into two quarks with the lightest neutralino, the final state topology is the same as that of the lighter chargino decay which consists of two jets with missing energy-momentum. On the other hand, the second lightest neutralino does not lead to one charged lepton with missing energy-momentum. The background from the pair production of the charged sleptons is discarded by requiring that one additional charged lepton be contained in at least one hemisphere.

The production $e^+e^- \to e^-\mu^+\omega_1^+\omega_1^-$ could also be induced by the generation-changing interaction in (8) through the pair production of different charginos ω_1 and ω_2. The heavier chargino decays into one charged lepton and a sneutrino, and this sneutrino decays into another charged lepton and a lighter chargino, $\omega_2^{+(-)} \to l_\alpha^{+(-)}\tilde{\nu}_a^{(*)}$, $\tilde{\nu}_a^{(*)} \to l_\beta^{-(+)}\omega_1^{+(-)}$. This process yields e and μ in one hemisphere and thus will be distinguishable from the previous process which leads to one charged lepton in each hemisphere. Therefore, the interaction could be examined independently by these two processes.

5 Summary

We have discussed the possibility of measuring the generation mixing for the interaction of charged leptons, sneutrinos, and charginos in future e^+e^- collision experiments. Numerical analyses have been made of the production and decay process $e^+e^- \to e^-\mu^+\omega_1^+\omega_1^-$. The experimental signal is given by an energetic charged lepton and two jets or a charged lepton with missing energy-momentum in each hemisphere. The experimental bound on the radiative decay $\mu \to e\gamma$ implies that the sneutrino mass difference and the mixing angle for the first two generations should be small. Under these constraints, the cross section of the process is larger than 0.05 fb in sizable regions of the parameter space. The SSM parameters for the chargino andneutralino sector affect non-trivially the cross section. Combined with information on charginos and neutralinos, the examination of the process will enable us to explore the generation mixing for the interaction. Or, on the contrary, the study of the generation mixing will give information on the SSM parameters.

Acknowledgements. This work is supported in part by the Grant-in-Aid for Scientific Research on Priority Areas (No. 16028204) from the Ministry of Education, Science and Culture, Japan.

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